

Part I (MULTIPLE CHOICE, NO CALCULATORS).

1. Compute $\int \left(2x^2 + \frac{3}{x} - 4 \sin x \right) dx$.

~~(a)~~ $\frac{1}{2} \left(2x^2 + \frac{3}{x} - 4 \sin x \right)^2 + C$

b/c wrong

~~(b)~~ $4x - \frac{3}{x^2} - 4 \cos x + C$

b/c $2x^2$

(c) $\frac{2}{3}x^3 + 3 \ln |x| - 4 \cos x + C$

(d) $\frac{2}{3}x^3 + 3 \ln |x| + 4 \cos x + C$

~~(e)~~ $\frac{2}{3}x^3 + 3 \ln |x| - 2 \sin^2 x + C$

2. Compute $\int_{-1}^1 x e^{x^2} dx$.

$u = x^2$

$du = 2x dx$

(a) 2

(b) $2e$

(c) e

(d) $\frac{1}{2}e^{\frac{1}{3}} - \frac{1}{2}e^{-\frac{1}{3}}$

(e) 0

$$\frac{1}{2} \int e^u du = \frac{1}{2} e^u du = \frac{1}{2} e^{x^2} \Big|_{-1}^1 = \frac{1}{2}(e - e)$$

3. Which of the following definite integrals gives the length of the curve $y = \sin x$ for $0 \leq x \leq \pi$?

(a) $\int_0^\pi \sqrt{1 + \cos^2 x} dx$

~~(b) $\int_0^\pi \cos x dx$~~

(c) $\int_0^\pi \sqrt{1 + \cos x} dx$

~~(d) $\int_0^\pi \sqrt{1 - \cos x} dx$~~

~~(e) $\int_0^\pi \sqrt{1 - \cos^2 x} dx$~~

4. Complete the square in the denominator of the integrand of the following indefinite integral and then compute the integral $\int \frac{1}{x^2 + 2x + 2} dx$.

(a) $\frac{1}{2} \tan^{-1}(x+1) + C$

$$(x^2 + 2x + 1) + 2 - 1 = (x+1)^2 + 1$$

~~(b) $\ln|x^2 + 2x + 2| + C$~~

$$\int \frac{1}{(x+1)^2 + 1} dx$$

~~(c) $\tan^{-1}(x^2 + 2x + 2) + C$~~

$$= \arctan(x+1)$$

~~(d) $(\tan^{-1} x)^2 + 1 + C$~~

(e) $\tan^{-1}(x+1) + C$

5. $\int_0^1 x e^x dx =$

(a) e

(b) $\frac{1}{2}$

(c) $2e$

(d) 1

(e) $\frac{1}{2}e$

$$u = x \quad du = 1$$

$$dv = e^x \quad v = e^x$$

$$x e^x \Big|_0^1 - \int_0^1 e^x = [1 \cdot e - 0 \cdot e^0] - e^x \Big|_0^1$$

$$= e - [e^1 - e^0] = 1$$

6. Which of the following statements is correct?

(a) $\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$ converges, and $\sum_{n=1}^{\infty} \frac{2n}{n^4 - 3}$ converges.

~~(b) $\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$ diverges, and $\sum_{n=1}^{\infty} \frac{2n}{n^4 - 3}$ diverges.~~

~~(c) $\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$ converges, and $\sum_{n=1}^{\infty} \frac{2n}{n^4 - 3}$ diverges.~~

~~(d) $\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$ diverges, and $\sum_{n=1}^{\infty} \frac{2n}{n^4 - 3}$ converges.~~

~~(e) $\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$ converges, and $\sum_{n=1}^{\infty} \frac{2n^4}{n^4 - 3}$ converges.~~

$$-\infty < \sum_{n=1}^{\infty} \frac{-1}{n^2} \leq \sum_{n=1}^{\infty} \frac{\sin n}{n^2} \leq \sum_{n=1}^{\infty} \frac{1}{n^2} < \infty$$

$$\sum_{n=1}^{\infty} \frac{2n^4}{n^4 - 3} \quad \text{but} \quad \frac{2n^4}{n^4 - 3} \rightarrow 2 \neq 0$$

7. $\int x^2 \cos(2x^3) dx =$

$u = 2x^3$

$du = 6x^2 dx$

~~(a) $\sin(2x^3) + x$~~

(b) $\frac{1}{6} \sin(2x^3) + C$

$\frac{1}{6} \int \cos u du = \frac{1}{6} \sin u$

~~(c) $\frac{1}{2} \sin(2x^3) + C$~~

~~(d) $\sin\left(\frac{1}{2}x^4\right) + C$~~

~~(e) $\frac{1}{3}x^3 \sin(2x^3) + C$~~

8. $\int \frac{\ln x}{x} dx =$

$u = \ln x$

$du = \frac{1}{x} dx$

(a) $\ln|x| + C$

(b) $\ln|\ln x| + C$

(c) $\frac{1}{2} \ln(x^2) + C$

(d) $\frac{1}{2} (\ln x)^2 + C$

$\int u du = \frac{1}{2} u^2 = \frac{1}{2} (\ln x)^2$

(e) $\ln\left(\frac{1}{2}x^2\right) + C$

9. $\int \frac{1}{2x+1} dx =$

$u = 2x+1$

$du = 2 dx$

(a) $\frac{1}{x^2+x} + C$

(b) $\ln|2x+1| + C$

$\frac{1}{2} \int \frac{1}{u} = \frac{1}{2} \ln u$

(c) $\frac{1}{2} \ln|2x+1| + C$

(d) $\frac{1}{(2x+1)^2} + C$

(e) $\ln|x^2+x| + C$

10. $\int \cos(x) e^{\sin(x)} dx =$

(a) $e^{\sin(x)} + C$

(b) $-e^{\sin(x)} + C$

(c) $-\sin e^{\cos x} + C$

(d) $-\sin x e^{\sin x} + C$

(e) $e^{\cos x} + C$

$u = \sin x$

$du = \cos x dx$

$\int e^u du = e^u$

11. Use $\int \frac{u^2}{\sqrt{a^2 - u^2}} du = -\frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{u}{a} \right) + C$ and substitution to compute $\int \frac{x^2}{\sqrt{5 - 4x^2}} dx$.

$\frac{u^2}{4} = x^2 \leftarrow u^2 = 4x^2 \quad a^2 = 5$

$u = 2x$

$du = 2 dx$

(a) $-x\sqrt{5 - 4x^2} + \frac{5}{2} \sin^{-1} \left(\frac{x}{5} \right) + C$

(b) $-\frac{x}{8} \sqrt{5 - 4x^2} + \frac{\sqrt{5}}{2} \sin^{-1} \left(\frac{x}{\sqrt{5}} \right) + C$

(c) $-\frac{x}{8} \sqrt{5 - 4x^2} + \frac{5}{16} \sin^{-1} \left(\frac{2x}{\sqrt{5}} \right) + C$

(d) $-\frac{x}{8} \sqrt{5 - 4x^2} + \frac{5}{8} \sin^{-1} \left(\frac{2x}{\sqrt{5}} \right) + C$

(e) $-\frac{x}{2} \sqrt{5 - 4x^2} + \frac{5}{2} \sin^{-1} \left(\frac{x}{\sqrt{5}} \right) + C$

$\int \frac{\left(\frac{u^2}{4}\right)}{\sqrt{a^2 - u^2}} \cdot \frac{1}{2} du = \frac{1}{8} \int \frac{u^2}{a^2 - u^2} du$
 $= -\frac{u}{16} \sqrt{a^2 - u^2} + \frac{a^2}{16} \sin^{-1} \left(\frac{u}{a} \right)$
 $= -\frac{2x}{16} \sqrt{5 - 4x^2} + \frac{5}{16} \sin^{-1} \left(\frac{2x}{\sqrt{5}} \right)$

12. Choose the definite integral that is equal to the following limit. $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(1 + \frac{i}{n} \right)^3 \frac{1}{n}$.

The Riemann Sum uses right-hand endpoints.

(a) $\int_1^2 (1+x)^3 dx \quad 2^3 \rightarrow 3^3$

(b) $\int_0^1 x^3 dx \quad 0^3 \rightarrow 1^3$

(c) $\int_1^2 x^3 dx \quad 1^3 \rightarrow 2^3$

(d) $\int_0^2 (1+x)^3 dx \quad 1^3 \rightarrow 3^3$

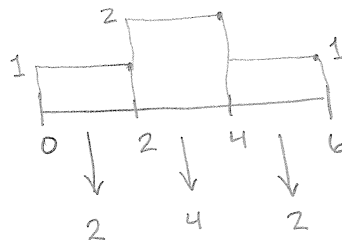
(e) $\int_0^2 x^3 dx \quad 0^3 \rightarrow 2^3$

at $i=0$, $1 + \frac{0}{n} = 1$
at $i=n$, $1 + \frac{n}{n} = 2$
 $1^3 \rightarrow 2^3$

Part II (MULTIPLE CHOICE, CALCULATORS ALLOWED).

1. A table of values of a function f is shown. We wish to estimate $\int_0^6 f(x) dx$. Do this by finding the Riemann sum for f on $[0, 6]$, using three equal subintervals and taking the sample points to be the right endpoints.

f	0	2	4	6
$f(x)$	5	1	2	1



- (a) 0
(b) 2
(c) 4
(d) 6
(e) 8
2. Compute $f''(x)$ where $f(x) = \int_0^x e^{t^2} dt$.

(a) e^{x^2}

(b) $2xe^{x^2}$

(c) xe^{x^2}

(d) e^{2x}

(e) $\frac{1}{2}e^{x^2}$

$$f'(x) = e^{x^2}$$

$$f''(x) = 2xe^{x^2}$$

3. Compute the average value of $\sin x$ on the interval $0 \leq x \leq \pi$.

(a) 2

(b) $\frac{1}{2} \sin x$

(c) $\frac{\pi}{2}$

(d) 0

(e) $\frac{2}{\pi}$

$$\frac{1}{\pi - 0} \int_0^{\pi} \sin x \, dx$$

$$\frac{1}{\pi} [-\cos x]_0^{\pi}$$

$$-\frac{1}{\pi} (-1 - 1) = \frac{2}{\pi}$$

4. What is the area of the region bounded by the curves $y = e^x$, $y = 0$ and the lines $x = 0$ and $x = \ln 2$?

(a) 1

(b) 2

(c) $\ln 2$ (d) e

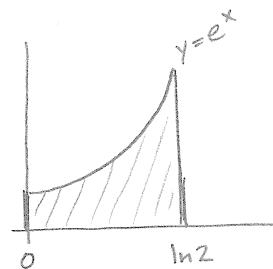
(e) 1.718

$$\int_0^{\ln 2} e^x dx$$

$$= e^x \Big|_0^{\ln 2}$$

$$= e^{\ln 2} - e^0$$

$$= 2 - 1$$



5. Find $\lim_{n \rightarrow \infty} \frac{\ln n}{n}$

(a) $-\infty$

(b) 0

(c) 1

(d) $+\infty$ (e) e

$$\lim_{n \rightarrow \infty} \frac{\ln n}{n} = \lim_{n \rightarrow \infty} \frac{(\frac{1}{n})}{1} = \lim_{n \rightarrow \infty} \frac{1}{n} = 0$$

6. Find the exact value (if the series converges) of the geometric series $\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n$. Be careful!

(a) $\frac{3}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$

(d) 1

(e) $\frac{2}{3}$

$$\sum_{n=1}^{\infty} \frac{1}{3} \left(\frac{1}{3}\right)^{n-1} = \frac{(\frac{1}{3})}{1 - \frac{1}{3}} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$$

- X 7. What is the best estimate of the error (to six decimal places) in using the first five terms of the alternating series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n \cdot 5^n}$ to approximate the value of the series.

(a) 0.000064

(b) 0

(c) 0.000008

(d) 0.000011

(e) 0.000400

8. Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{x^n}{n \cdot 2^n}$.

(a) 1

(b) 2

(c) $\frac{1}{2}$

(d) 4

(e) The series diverges.

$$\left| \frac{x^{n+1}}{(n+1)2^{n+1}} \cdot \frac{n2^n}{x^n} \right| = \left| \frac{xn}{2(n+1)} \right| = |x| \cdot \frac{n}{2n+2} \xrightarrow{n \rightarrow \infty} \frac{|x|}{2} < 1$$

$$|x| < 2$$

9. Compute the power series representation centered at $a = 0$ for the function $\frac{1}{(1-x)^2}$.

Note that $\frac{1}{(1-x)^2} = \frac{d}{dx} \left[\frac{1}{1-x} \right]$.

(a) $\sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1}$

(b) $\sum_{n=0}^{\infty} x^{2n}$

(c) $\sum_{n=1}^{\infty} 2x^n$

(d) $\sum_{n=1}^{\infty} nx^{n-1}$

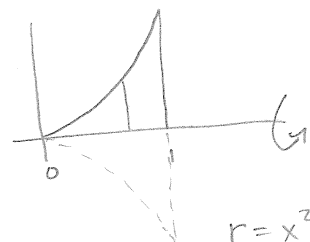
(e) $\sum_{n=1}^{\infty} \frac{x^{n+1}}{n+1}$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + x^4 + \dots$$

$$\frac{d}{dx} \left(\frac{1}{1-x} \right) = \sum \frac{d}{dx} (x^n) = 1 + 2x + 3x^2 + 4x^3 + \dots$$

$$= \sum nx^{n-1}$$

10. Find the volume generated when the area bounded by $y = x^2$, $y = 0$ and the lines $x = 0$ and $x = 1$ is rotated about the x -axis.

(a) $\frac{\pi}{5}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{1}{5}$ (e) $\frac{1}{3}$ 

$$A(x) = x^4 \pi$$

$$\pi \int_0^1 x^4 dx = \pi \frac{x^5}{5} \Big|_0^1 = \frac{\pi}{5}$$

11. The series $\sum_{n=0}^{\infty} (-1)^n \frac{x^{4n}}{n!}$ represents which of the following functions.

(a) e^{-4x}

(b) $\sin 4x$

(c) $\cos 4x$

(d) e^{-x^4}

(e) $\frac{1}{1+x^4}$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$e^{-x^4} = \sum_{n=0}^{\infty} \frac{(-x^4)^n}{n!} = \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n}}{n!}$$

12. Find A and B in the partial fraction $\frac{x+2}{(x+3)(x-2)} = \frac{A}{x+3} + \frac{B}{x-2}$.

(a) $A = -3, B = 2$

(b) $A = -\frac{1}{5}, B = -\frac{4}{5}$

(c) $A = \frac{1}{2}, B = -\frac{1}{2}$

(d) $A = \frac{4}{5}, B = \frac{1}{5}$

(e) $A = \frac{1}{5}, B = \frac{4}{5}$

$$x+2 = A(x-2) + B(x+3)$$

$x=2$ $4 = 5B$, $B = 4/5$

$x=-3$ $-1 = -5A$, $A = 1/5$

13. If the improper integral $\int_0^{\infty} x e^{-x^2} dx$ converges, its value is

(a) 0

(b) $\frac{1}{2}$

(c) 1

(d) e

(e) The integral diverges

$$u = -x^2$$

$$du = -2x dx$$

$$-\frac{1}{2} \int_0^{\infty} e^u du = -\frac{1}{2} e^u = -\frac{1}{2} e^{-x^2} \Big|_0^{\infty}$$

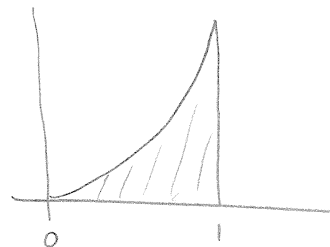
$$= -\frac{1}{2} e^{-\infty} + \frac{1}{2} e^0$$

$$= 0 + \frac{1}{2}$$

Part III (FREE RESPONSE, CALCULATORS ALLOWED).

1. (a) [4 points] Compute the area of the region in the first quadrant, bounded by the graphs of $y = x^2$, $y = 0$, and $x = 1$.

$$\int_0^1 x^2 dx = \left. \frac{x^3}{3} \right|_0^1 = \frac{1}{3}$$



- (b) [6 points] Find the coordinates $(\bar{x}, \bar{y})$ of the centroid of the area in the first quadrant bounded by the graphs of $y = x^2$, $y = 0$, and $x = 1$.

$$\bar{x} = \frac{1}{A} \int_a^b x f(x) dx$$

$$= 3 \int_0^1 x \cdot x^2 dx$$

$$= 3 \int_0^1 x^3 dx$$

$$= \left. \frac{3x^4}{4} \right|_0^1$$

$$= \frac{3}{4}$$

$$\bar{y} = \frac{1}{A} \int_a^b \frac{1}{2} [f(x)]^2 dx$$

$$= 3 \int_0^1 \frac{1}{2} x^4 dx$$

$$= \frac{3}{2} \cdot \left. \frac{x^5}{5} \right|_0^1$$

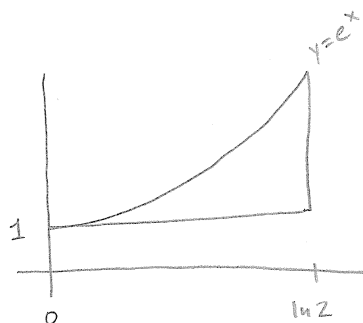
$$= \frac{3}{2} \cdot \frac{1}{5}$$

$$= \frac{3}{10}$$

$$\text{centroid} = \left(\frac{3}{4}, \frac{3}{10} \right) = (.75, .3)$$

2. Each part of this problem refers to the region R bounded by $y = e^x$, $y = 1$ and the lines $x = 0$ and $x = \ln 2$.

In each part, give the appropriate definite integral, but DO NOT evaluate the integrals.



- (a) [2.5 points] Give the definite integral needed to compute the volume generated when the region R is rotated about the x -axis.

$$V = \pi \int_0^{\ln 2} (e^{2x} - 1) dx$$

$$\begin{aligned} R &= e^x \\ r &= 1 \\ A(x) &= R^2\pi - r^2\pi \\ &= (e^{2x} - 1)\pi \end{aligned}$$

- (b) [2.5 points] Give the definite integral needed to compute the volume generated when the region R is rotated about the y -axis. You must use washers for this part.

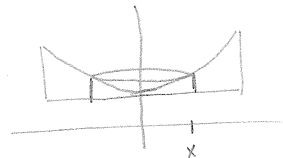
$$V = \pi \int_1^2 ((\ln 2)^2 - (\ln y)^2) dy$$

$$\begin{aligned} R &= \ln 2 \\ r : y &= e^x \\ x &= \ln y \\ A(y) &= R^2\pi - r^2\pi \\ &= ((\ln 2)^2 - (\ln y)^2)\pi \end{aligned}$$

$$y = e^{\ln 2} = 2$$

- (c) [2.5 points] Give the definite integral needed to compute the volume generated when the region R is rotated about the y -axis. You must use shells for this part.

$$V = 2\pi \int_0^{\ln 2} x(e^x - 1) dx$$



$$\boxed{} \left. \vphantom{\int} \right\} h = e^x - 1$$

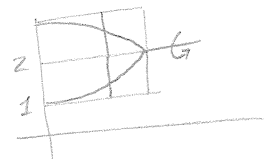
$$2\pi r = 2\pi x$$

Cylinder :

$$2\pi x (e^x - 1) \Delta x$$

- (d) [2.5 points] Give the definite integral needed to compute the volume generated when the region R is rotated about the line $y = 2$.

$$V = \pi \int_0^{\ln 2} 1 - (2 - e^x)^2 dx$$



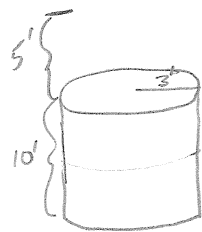
$$R = 1$$

$$r = 2 - e^x$$

$$A(x) = R^2 \pi - r^2 \pi$$

$$= (1 - (2 - e^x)^2) \pi$$

$$1000 \frac{\text{kg}}{\text{m}^3} \cdot \left(\frac{\text{m}}{100\text{cm}}\right)^3 \cdot \left(\frac{2.54\text{cm}}{1\text{in}}\right)^3 \cdot \left(\frac{12\text{in}}{1\text{ft}}\right)^3$$



3. In this problem, we will compute the work done in pumping water out of a right circular cylindrical tank. The height of the tank is 10 feet and its radius is 3 feet. The tank is sitting on its circular base and is full of water (density 62.5 lbs/cu.ft.). The water is to be pumped to a level 5 feet above the top of the tank.

← $g \times \text{density}$
"weight"

- (a) [8 points] Set up the definite integral needed to compute the work done. [the value of the integral will be computed in part (b).]

$$V_i = r^2 \pi \Delta x = 3^2 \pi \Delta x = 9\pi \Delta x \quad \text{ft}^3 \quad \text{---} \quad \text{disk} \quad \Delta x$$

$$F_i = V_i \cdot \text{weight} = 9\pi(62.5) \Delta x \quad \text{lbs.}$$

Force
in lbs

$$W_i = 9\pi(62.5) \Delta x \cdot x_i$$

$$x_i = \text{distance} \rightarrow 5 \text{ to } 15$$

$$\int_5^{15} 9\pi(62.5)x \, dx$$

- (b) [2 points] Evaluate the definite integral in part (a) to get the numerical answer to the work done.

$$562.5\pi \int_5^{15} x \, dx = 562.5\pi \cdot \frac{x^2}{2} \Big|_5^{15}$$

$$= 281.25\pi (225 - 25)$$

$$= 56250\pi$$

$$= 176714.5868$$

4. (a) [3 points] The partial sums of the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ are given by $S_n = \sum_{i=1}^n \frac{1}{i^2}$.

Compute

S_1, S_2, S_3, S_4 and S_5 .

$$S_1 = \frac{1}{1^2} = 1$$

$$S_2 = \frac{1}{1^2} + \frac{1}{2^2} = 1 + \frac{1}{4} = \frac{5}{4}$$

$$S_3 = \frac{1}{1} + \frac{1}{4} + \frac{1}{9} = \frac{5}{4} + \frac{1}{9} = \frac{45+4}{36} = \frac{49}{36}$$

$$S_4 = \frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} = \frac{49}{36} + \frac{1}{16} = \frac{205}{144}$$

$$S_5 = \frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} = \frac{205}{144} + \frac{1}{25} = \frac{5269}{3600}$$

- (b) [3 points] In terms of limits, what is the relation between the series $\sum_{n=1}^{\infty} a_n$ and

the sequence of partial sums $S_n = \sum_{i=1}^n a_i$?

$$\sum_{n=1}^{\infty} a_n = \lim_{n \rightarrow \infty} S_n$$

- (c) [4 points] Given a simple example of a series $\sum_{n=1}^{\infty} a_n$ that diverges but $\lim_{n \rightarrow \infty} a_n = 0$.

$$\sum_{n=1}^{\infty} \frac{1}{n} \quad \text{diverges}$$

$$\text{but } \lim_{n \rightarrow \infty} \frac{1}{n} = 0$$

5. This problem is concerned with the power series $\sum_{n=1}^{\infty} (-1)^n \frac{(x+2)^n}{n \cdot 2^n}$.

- (a) [5 points] Find the interval of convergence for this series. Don't forget to check the endpoints of the interval.

$$\lim_{n \rightarrow \infty} \left| \frac{\frac{(-1)^{n+1} (x+2)^{n+1}}{(n+1) 2^{n+1}}}{\frac{(-1)^n (x+2)^n}{n 2^n}} \right| = \lim_{n \rightarrow \infty} \left| \frac{(x+2)^{n+1}}{(n+1) 2^{n+1}} \cdot \frac{n \cdot 2^n}{(x+2)^n} \right|$$

$$= \lim_{n \rightarrow \infty} \left| \frac{(x+2) n}{(n+1) \cdot 2} \right| = |x+2| \lim_{n \rightarrow \infty} \frac{n}{2(n+1)} = \frac{|x+2|}{2} < 1$$

$$|x+2| < 2$$

$$-2 < x+2 < 2$$

$$-4 < x < 0$$

$$x=0, \quad \sum (-1)^n \frac{(0+2)^n}{n 2^n} = \sum \frac{(-1)^n}{n} \quad C$$

$$x=-4, \quad \sum (-1)^n \frac{(-2)^n}{n 2^n} = \sum \frac{1}{n} \quad D$$

$$\text{interval} = (-4, 0]$$

- (b) [3 points] Approximate the value of the series when $x = -1.9$ using the first five terms.

$$\begin{aligned}
 & (-1) \frac{(-1.9+2)}{2} + \frac{(-1.9+2)^2}{2 \cdot 4} - \frac{(2-1.9)^3}{3 \cdot 8} + \frac{(2-1.9)^4}{4 \cdot 16} - \frac{(2-1.9)^5}{5 \cdot 32} \\
 & = -.05 + .00125 - \text{circled term} + \text{circled term} - \text{circled term} \\
 & = -.0487901667
 \end{aligned}$$

X

- (c) [3 points] Estimate the error in the approximation in part (b). Note that the series is an alternating series when $x = -1.9$.

6. (a) [3 points] Use the integral test to show that the series $\sum_{n=1}^{\infty} \frac{1}{n^3}$ converges. I.e., show

that the improper integral $\int_1^{\infty} \frac{1}{x^3} dx$ converges.

$$\int_1^{\infty} x^{-3} dx = \frac{x^{-2}}{-2} = -\frac{1}{2x^2} \Big|_1^{\infty} = -\frac{1}{2} + \frac{1}{\infty} = -\frac{1}{2}$$

- (b) [4 points] Let $S = \sum_{n=1}^{\infty} \frac{1}{n^3}$, $S_n = \sum_{k=1}^n \frac{1}{k^3}$ and $f(x) = \frac{1}{x^3}$. Given that $S_n + \int_{n+1}^{\infty} f(x) dx \leq S \leq S_n + \int_n^{\infty} f(x) dx$, compute A and B (to six decimal places) so that $A \leq S \leq B$ for $n = 10$.

$$\begin{aligned} A = S_{10} + \int_{11}^{\infty} \frac{1}{x^3} dx &= \sum_{k=1}^{10} \frac{1}{k^3} + \int_{11}^{\infty} \frac{1}{x^3} dx = 1.19753198567 \\ &\quad + .004132231405 \\ &= 1.20166421708 \end{aligned}$$

$$\begin{aligned} B = S_{10} + \int_{10}^{\infty} \frac{1}{x^3} dx &= \sum_{k=1}^{10} \frac{1}{k^3} + \int_{10}^{\infty} \frac{1}{x^3} dx = 1.19753198567 \\ &\quad + .005 \\ &= 1.20253198567 \end{aligned}$$

X(c) [3 points] Let $R_n = S - S_n$ (remainder after n terms). Given that $R_n \leq \int_n^\infty f(x) dx$, find the minimum number of terms ($n=?$) so that $R_n \leq 0.0005$.